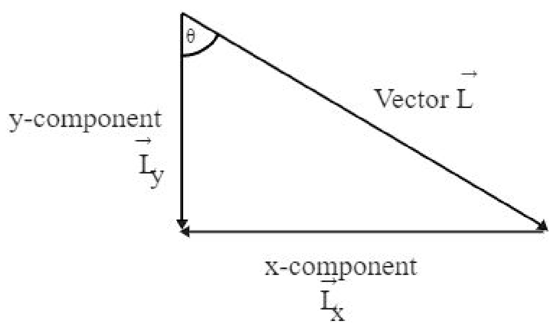
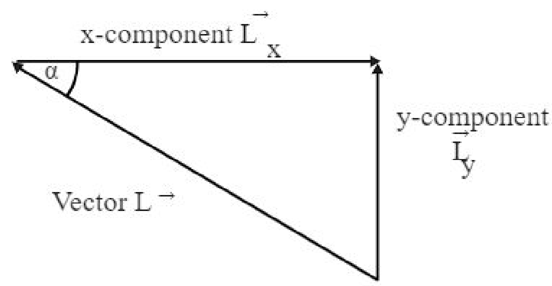
Vector Addition Using Components

In order to analyze and solve problems related to vectors, you will often need to add and subtract them. To be successful at adding and subtracting vectors you must obey the rule that vectors can only be added, (or subtracted) if they lie along the same axis.

Before you add or subtract vectors you must be able to *resolve* a vector into its components. In this section you will learn what a vector component is and how to resolve it.

A vector that does not lie directly along a horizontal or vertical axis can be thought of as being composed of two parts, called **components**.



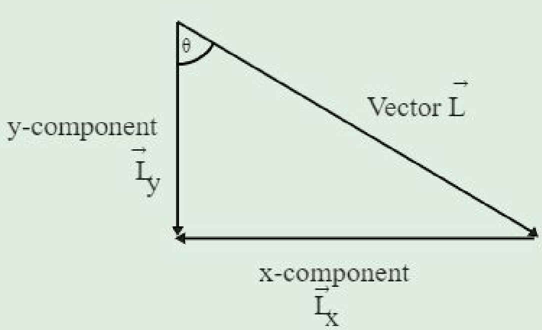


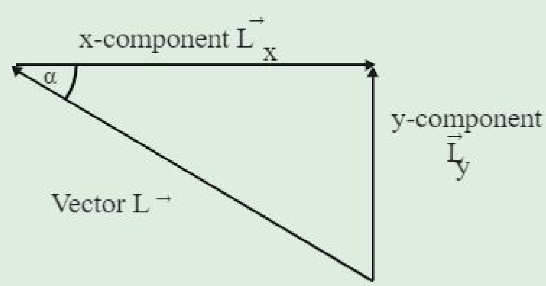
In the above two images you can see a vector with a magnitude of "*L*".

This vector is composed of a horizontal part – the ***x-component*** and a vertical part – the ***y-component***. When these two vectors are connected head to tail, they form a 90° angle.

In addition, when a resultant vector is drawn from the initial tail to the final head of these two component vectors, a complete right-angled triangle is formed. Because a vector and its two components make up a right-angled triangle, trigonometric functions (sine, cosine, and tangent) can be used to mathematically resolve a vector into its components.

Here's a quick review of the trigonometric functions that are used to mathematically resolve a vector into its components





Note: this quick review is specific for the right-angled triangle above to the left containing the angle θ

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{L_x}{L}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{L_y}{L}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{L_x}{L_y}$$

Where θ is the known angle of the vector, the "opposite side" is the side opposite the known angle, and the "adjacent side" is the side helping to make up the angle.

Example 1

Resolve a velocity vector of 25 m/s [E 45° N] into its components.

For the x-component use the cosine function:

$$\cos\theta = \frac{v_x}{v}$$

Rearranging and substituting:

$$v_x = v\cos\theta$$

$$v_x = (25\text{m/s})(\cos 45^\circ)$$

$v_x = 17.7\text{m/s}$ (extra significant digit)

$$v_x = 18\text{m/s}$$

For the y-component use the sine function:

$$\sin\theta = \frac{v_y}{v}$$

Rearranging and substituting:

$$v_y = v\sin\theta$$

$$v_y = (25\text{m/s})(\sin 45^\circ)$$

$v_y = 17.7\text{m/s}$ (extra significant digit)

$$v_y = 18\text{m/s}$$